

NAME:

Math 150 Practice Exam 1.3

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. Given that $\lim_{x \rightarrow -2} f(x) = 4$, $\lim_{x \rightarrow -2} g(x) = -3$, and $\lim_{x \rightarrow -2} h(x) = 0$ find

a) $\lim_{x \rightarrow -2} [f(x) - 2g(x)]$ [5 pts]

b) $\lim_{x \rightarrow -2} \frac{\sqrt{f(x) \cdot [g(x)]^2}}{h(x) + f(x)}$ [5 pts]

2. Use the squeeze theorem to evaluate $\lim_{x \rightarrow 0^+} \sqrt{x} e^{\sin(\pi/x)}$ [10 pts]

3. Evaluate $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$ [10 pts]

4. Find an equation of the tangent line to the curve $y = \frac{x \sin(x)}{1+x}$ at the point $(0, 0)$. [10 pts]

5. Find the derivative of the function $f(x) = \sqrt{1-2x}$ using the definition of the derivative at the point $x = a$. [10 pts]

6. Let $f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$. Determine the points where f is discontinuous.

Justify your answer.

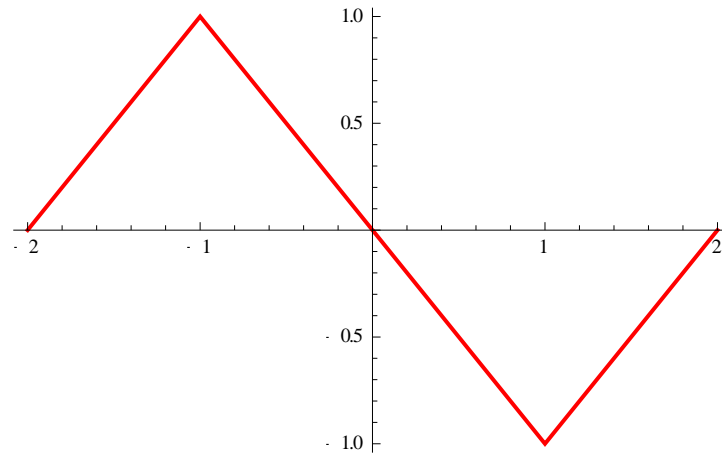
[10 pts]

7. Let $f: [0, 1] \rightarrow (0, 1)$ be a continuous function such that $0 < f(x) < 1$ for all $x \in [0, 1]$. Prove that the equation $f(x) = x^2$ has a solution for at least one $x \in [0, 1]$. [10 pts]

8. Let $a > 0$ be a positive real number. Define $f(x) = \begin{cases} x^2 & \text{if } x < a \\ 3x & \text{if } x \geq a \end{cases}$.

What is the value of a if f is continuous on the entire real number line? [10 pts]

9. The graph of the function $y = f(x)$ is displayed below



Draw the graph of $y = f'(x)$.

[10 pts]

10. Evaluate $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta}$

[10 pts]

Extra-Credit

11. Prove by means of a delta-epsilon argument that $\lim_{x \rightarrow 2} (x^2 - 4x + 5) = 1$ [10 pts]

12. Calculate $\lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos(x-h)}{h}$ [10 pts]